Partially polarized beams in nonlinear Thomson scattering

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We discuss the electromagnetic potential of partially polarized light, which includes as a special case linearly polarized lasers, and the radiation of free electrons induced by such beams. A classical electrodynamics calculation is performed in order to obtain the low-order harmonic cross sections at low beam intensities. According to the correspondence principle for radiation processes in the Thomson limit, we find agreement between our results and those of quantum electrodynamics. The implications of this correspondence for processes at arbitrary intensities, relativistic electrons, and pulsed beams are also underlined.

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Classical and quantum electrodynamics (QED) must agree in the Thomson limit (TL), e.g., a rest electron scattering a low frequency and intensity monochromatic plane wave. A consistent physical and theoretical understanding of the interaction of light beams with free electrons, within classical and quantum treatments, is thus not only of importance in its own right but also as a necessary piece for further comprehending the more complex scattering of electrons with ultrahigh-power lasers. It is also relevant to topics of current physical interest from both theoretical [1–4] and practical [1,5,6] points of view. Among possible practical applications of high intensity lasers [7] brought into collision with electrons would be the design of a compact source of short pulse x rays that utilizes the Compton-like backscattering of x rays from such lasers [1,5,6]. X rays originating in this way would generally be produced in certain states of partial polarization.

We stress that classical calculations [1,3,8] for arbitrary field intensities, laser pulse shapes, and relativistic electrons, should go over into the standard classical-QED correspondence in the TL. For the higher harmonic (or multiphoton) cross section this is a nontrivial check since it is not uncommon, as illustrated below, to find agreement for the fundamental harmonic Thomson cross section whereas the corresponding classical second harmonic rate disagrees with QED in the TL.

For ordinary Compton scattering, as is well known, classical and QED results are identical in the TL for all kinds of incident beams. However, for the nonlinear higher harmonic processes such agreement was demonstrated only for incident circularly (all harmonics) [9] and linearly (second harmonic) [2,10] polarized light. The case of partially polarized light, perhaps the most common form of light everywhere in nature, is clearly of great physical interest. In particular, unpolarized or natural light offers one example of agreement at the level of the Thomson cross section that contrasts the disparate results so far obtained for second harmonic radiation [4,10,11]. We shall show below that the second harmonic classical cross section for partially polarized light agrees with the QED result [4] in the TL.

We describe the QED process as the simultaneous absorption of two or more photons, with equal or distinct polarizations, together with the emission of a single final photon [4]. The systematic analysis of Feynman graph amplitudes gives an unequivocal answer in this case. On the other hand, in classical electromagnetism, it is the motion of the electron in the wave that determines the radiation given off. Thus, a physically relevant question here refers to the form of a general vector potential that, after allowing for random fluctuations of the field, describes correctly the scattered radiation out of a partially polarized wave. In the following, such a general form of the vector potential is introduced valid for partially polarized incident waves.

The orbit of an electron in a plane monochromatic wave depends on the potential $\mathbf{A}$ representing the wave and determines the emitted radiation. For beams in definite states of polarization, $\mathbf{A}$ has known functional forms. A number of theoretical calculations include the Schott cross section for circularly polarized light [12,13], as well as other results for linear, circular, and elliptically polarized incident beams [1,8,11]. In the more general case of light in a state of partial polarization, though a common physical situation, it is not immediately clear *a priori* what is the functional form of the vector potential representing the wave.

As we know, intensity fluctuations of partially polarized light can be described by assuming a superposition of two independent beams linearly polarized parallel to the x and y axis. The instantaneous cycle-average total intensity is thus $\tilde{I}=\tilde{I}_x+\tilde{I}_y$, where $\tilde{I}_\lambda$ is the intensity of the λ-polarized beam. The above statement does not pay attention to the radiation emitted by an electron in this wave. It does, however, suggest a general vector potential superposition of two linearly polarized waves. We note that in the radiation gauge the potential must involve only the two independent amplitudes $A_{0\lambda}, \tilde{I}_\lambda \propto A_{0\lambda}^2$, along each axis $\lambda=x,y$.

Let $k^\mu=(\omega/c,\mathbf{k})$ be the wave vector of the incident beam which propagates along the positive z axis. The monochromatic potential $\mathbf{A}(\zeta)$ we seek must be expressed as the superposition $\mathbf{A}=\mathbf{A}_1+\mathbf{A}_2$ of the two independent potentials with arbitrary linear polarization

$$\mathbf{A}_1=(\hat{\varepsilon}_x A_{0x}+\hat{\varepsilon}_y A_{0y})\cos\zeta \;, \tag{1a}$$

$$\mathbf{A}_2=(-\hat{\varepsilon}_x A'_{0x}+\hat{\varepsilon}_y A'_{0y})\sin\zeta\ , \tag{1b}$$

where $\zeta=k\cdot x=\omega(t-z/c)$. Of the simpler choices that reduce such superposition to a potential containing only two independent amplitudes along each axis (say $A_{0\lambda}$), (i) $A'_{0x}=A_{0x}$, $A'_{0y}=A_{0y}$, and (ii) $A'_{0x}=A'_{0y}=0$ give elliptic and linear polarization which do not correspond to the case of a partially polarized beam. If, on the other hand, we choose $A'_{0x}=0$, $A'_{0y}=A_{0y}$, or

$$A=\hat{\varepsilon}_x A_{0x}\cos\zeta+\hat{\varepsilon}_y A_{0y}(\cos\zeta+\sin\zeta)\ , \tag{2}$$

we find a more general potential which is not linearly nor elliptically polarized. Notice that it can also be interpreted as the superposition of (1a) with the elliptically polarized wave $\mathbf{A}^{(+)}=\hat{\varepsilon}_x A_{0x}\cos\zeta+\hat{\varepsilon}_y A_{0y}\sin\zeta$, or as the superposition of two linearly polarized potentials along the x and y axis given by each term in (2), respectively. In any case Eq. (2) represents a polarization vector that changes with time at all points of the wave, i.e., not according with linear or elliptic polarization. We shall assume this form in our calculations below. Ultimately, Eq. (2) will be justified *a posteriori* by our results.

The solution for the orbit of a relativistic electron moving in the field of a monochromatic plane wave of arbitrary intensity is, of course, quite familiar from preceding discussions in the literature [14,15]. Our interest here is in the solution for the periodic motion of an electron, at rest on the average, whose trajectory is $\tilde{\mathbf{r}}=(\tilde{x},\tilde{y},\tilde{z})$. We write (a bar means cycle average) [15]

$$\tilde{x}=-\frac{e}{m_*c\omega}\int A_x d\zeta,\quad \tilde{y}=-\frac{e}{m_*c\omega}\int A_y d\zeta\ ,$$
$$\tilde{z}=(e^2/2m_*^2c^3\omega)\int(A^2-\overline{A^2})d\zeta\ , \tag{3a}$$

where the mass m_* of the electron inside the wave (2) and the field-intensity parameter τ are

$$m_*^2=m^2(1+\tau^2),\quad \tau^2=(e^2\overline{A^2}/m^2c^4)=\tau_x^2+2\tau_y^2\ ,$$
$$\tau_\lambda=(eA_{0\lambda}/\sqrt{2}mc^2)\ ,\quad \lambda=x,y\ . \tag{3b}$$

Substitution of (2) in (3a) gives the normalized orbit $\mathbf{r}=\omega\tilde{\mathbf{r}}/c=(x,y,z)$ and its ζ derivative $\mathbf{v}$

$$x=-x_0\sin\zeta,\quad y=y_0(\cos\zeta-\sin\zeta)\ ,$$
$$z=\tfrac{1}{8}(x_0^2\sin2\zeta-2y_0^2\cos2\zeta)\ , \tag{3c}$$

$$v_x=-x_0\cos\zeta,\quad v_y=-y_0(\sin\zeta+\cos\zeta)\ ,$$
$$v_z=\tfrac{1}{4}(x_0^2\cos2\zeta+2y_0^2\sin2\zeta)\ , \tag{3d}$$

with

$$x_0=\frac{\sqrt{2}\tau_x}{(1+\tau_x^2+2\tau_y^2)^{1/2}}\ ,\quad y_0=\frac{\sqrt{2}\tau_y}{(1+\tau_x^2+2\tau_y^2)^{1/2}}\ . \tag{4}$$

Notice that the electron trajectory in (3c) for the particular case of linear polarization along the x axis ($\tau_y=0$) assumes a familiar form previously used in the literature [8,16].

In order to calculate the l-harmonic power P_l radiated per cycle into solid angle $d\Omega$ by an electron in a periodic orbit such as (3c), we use the well known expression [8,17]

$$dP_l/d\Omega=(e^2l^2\omega^2/8\pi^3c)\sum_{j=1,2}|S_j^{(l)}|^2\ , \tag{5a}$$

$$S_j^{(l)}=\int_0^{2\pi}[\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{v})]_j\exp\{il[\zeta+z-\hat{\mathbf{n}}\cdot\mathbf{r}]\}d\zeta\ , \tag{5b}$$

where $\hat{\mathbf{n}}=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$ is a constant unit vector defining the solid angle of the emitted radiation. Note the integral (5b) is in terms of the variable ζ (instead of time t), and that the polarization vector $\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{v})$ is normal to $\hat{\mathbf{n}}$ so that it can be resolved into the two components $\hat{\varepsilon}_1=(\cos\theta\cos\phi,\cos\theta\sin\phi,-\sin\theta)$, and $\hat{\varepsilon}_2=(-\sin\phi,\cos\phi,0)$. The final polarization is not detected, hence the sum in (5a).

From the above relations the components of this vector are $[\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{v})]_1=-v_x\cos\theta\cos\phi-v_y\cos\theta\sin\phi+v_z\sin\theta$ and $[\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{v})]_2=v_x\sin\phi-v_y\cos\phi$, so that

$$[\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{v})]_1=(x_1+y_1)\cos\zeta+y_1\sin\zeta$$
$$+(\sin\theta/4)(x_0^2\cos2\zeta+2y_0^2\sin2\zeta)\ , \tag{6a}$$

$$[\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{v})]_2=(-x_0\sin\phi+y_0\cos\phi)\cos\zeta+y_0\cos\phi\sin\zeta\ , \tag{6b}$$

where $x_1=x_0\cos\theta\cos\phi$, $y_1=y_0\cos\theta\sin\phi$. Similarly, in the argument of the exponential function of (5b) we shall need

$$z-\hat{\mathbf{n}}\cdot\mathbf{r}=(x'_1+y'_1)\sin\zeta-y'_1\cos\zeta$$
$$+\frac{(1-\cos\theta)}{8}(x_0^2\sin2\zeta-2y_0^2\cos2\zeta)\ , \tag{7}$$

with $x'_1=x_0\sin\theta\cos\phi$, $y'_1=y_0\sin\theta\sin\phi$.

The l-harmonic cross section is defined by

$$d\sigma_l/d\Omega=1/I\langle dP_l/d\Omega\rangle\ , \tag{8}$$

where $I=\langle\tilde{I}\rangle=I_x+I_y$, $I_\lambda=\langle\tilde{I}_\lambda\rangle$ $(\lambda=x,y)$, represent average intensities, and $\tilde{I}_x=\omega^2A_{0x}^2/(8\pi c)$, $\tilde{I}_y=\omega^2A_{0y}^2/(4\pi c)$ the cycle-average intensity components defined above.

The previous discussion considers fields of arbitrary intensity; we now specialize to the low intensity regime $\tau_\lambda\ll1$, thus $x_0,y_0\ll1$ and $z-\hat{\mathbf{n}}\cdot\mathbf{r}\ll1$. In this case the ordinary Thomson cross section is of $O(\tau_\lambda^2,\tau_x\tau_y)$, then clearly it requires only the approximations $\exp[i(z-\hat{\mathbf{n}}\cdot\mathbf{r})]\approx1$ and $[\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{v})]_1\approx(x_1+y_1)\cos\zeta+y_1\sin\zeta$ in the integrand of (5b). Hence,

$$S_1^{(1)}=\int_0^{2\pi}e^{i\zeta}[(x_1+y_1)\cos\zeta+y_1\sin\zeta]d\zeta$$
$$=\pi(x_1+y_1+iy_1)\ , \tag{9a}$$

$$S_2^{(1)}=\int_0^{2\pi}e^{i\zeta}[(-x_0\sin\phi+y_0\cos\phi)\cos\zeta+y_0\cos\phi\sin\zeta]d\zeta$$
$$=\pi(-x_0\sin\phi+y_0\cos\phi+iy_0\cos\phi)\ . \tag{9b}$$

The cross section for partially polarized incident beams and final polarization not detected is thus, after substitution of (9) in (5a) and (8),

$$\frac{d\sigma_1}{d\Omega}=r_0^2[g_x^{(1)}(0)(\cos^2\theta\cos^2\phi+\sin^2\phi)$$
$$+g_y^{(1)}(0)(\cos^2\theta\sin^2\phi+\cos^2\phi)]\ , \tag{10a}$$

where $r_0=e^2/(mc^2)$ is the classical electron radius. Moreover, to obtain (10a) we have used $\langle A_{0x}A_{0y}\rangle=\langle A_{0x}\rangle\langle A_{0y}\rangle=0$, assumed for fields whose intensity fluctuations are independent, and defined the classical degrees of first-order coherence as

$$g_x^{(1)}(0)=\frac{I_x}{I}=\tfrac{1}{2}(1+P)\ ,\quad g_y^{(1)}(0)=\frac{I_y}{I}=\tfrac{1}{2}(1-P)\ ,\qquad (10b)$$

with $P=(I_x-I_y)/I$ the degree of polarization.

It is interesting to note that (a) both potentials in Eqs. (1), the above superpositions (i) and (ii) of (1), and the potential $\mathbf{A}^{(+)}$ defined before, lead also to the Thomson cross section (10); and (b) the QED result in the TL is identical to (10a) and (10b) provided intensities are substituted by expectation values of the number of photons in the beam. However, in terms of P, the quantum degrees of coherence are the same as those in (10b). Both theories agree for all beams in this case, as previously mentioned.

We now turn to the next order or $l=2$ cross section to $O(\tau_\lambda^4,\tau_x^2\tau_y^2,\tau_x^3\tau_y,\tau_x\tau_y^3)$ in the low intensity regime. It is clear that only terms to $O(\tau_\lambda)$ need be retained in the expansion of $\exp[2i(z-\hat{\mathbf{n}}\cdot\mathbf{r})]$ in (5b). The integral can now be evaluated by inserting (6) and (7) in (5b),

$$S_1^{(2)}=\int_0^{2\pi}e^{2i\zeta}\left[(x_1+y_1)\cos\zeta+y_1\sin\zeta+\frac{\sin\theta}{4}(x_0^2\cos2\zeta+2y_0^2\sin2\zeta)\right]\{1+2i[(x_1'+y_1')\sin\zeta-y_1'\cos\zeta]\}d\zeta\ ,\qquad (11a)$$

$$S_2^{(2)}=\int_0^{2\pi}e^{2i\zeta}[(-x_0\sin\phi+y_0\cos\phi)\cos\zeta+y_0\cos\phi\sin\zeta]\{1+2i[(x_1'+y_1')\sin\zeta-y_1'\cos\zeta]\}d\zeta\ ,\qquad (11b)$$

with the result to the required order

$$S_1^{(2)}=\pi\{(x_0^2/4)\sin\theta-(x_1'+y_1')(x_1+y_1)+y_1y_1'-i[(x_1'+y_1')y_1+y_1'(x_1+y_1)-(y_0^2/2)\sin\theta]\}\ ,\qquad (12a)$$

$$S_2^{(2)}=\pi\{(x_1'+y_1')x_0\sin\phi-y_0x_1'\cos\phi+i[x_0y_1'\sin\phi-y_0(x_1'+2y_1')\cos\phi]\}\ .\qquad (12b)$$

Using (12a) and (12b) in (5a) and (8) we can now derive the second harmonic cross section for partially polarized incoming light and unpolarized emitted radiation. Thus,

$$(d\sigma_2/d\Omega)=(\eta r_0/\sqrt{2})^2 16\sin^2\theta\{g_{xx}^{(2)}(0)[(\cos\theta\cos^2\phi-\tfrac{1}{4})^2+\tfrac{1}{4}\sin^2 2\phi]$$
$$+g_{yy}^{(2)}(0)[(\cos\theta\sin^2\phi-\tfrac{1}{4})^2+\tfrac{1}{4}\sin^2 2\phi]+g_{xy}^{(2)}(0)(\cos^2 2\phi+\cos^2\theta\sin^2 2\phi)\}\ .\qquad (13)$$

As before we have used for the two orthogonal independent beam components $\langle A_{0x}A_{0y}^3\rangle=\langle A_{0x}\rangle\langle A_{0y}^3\rangle=\langle A_{0x}^3A_{0y}\rangle=\langle A_{0x}^3\rangle\langle A_{0y}\rangle=0$, and exhibited the classical degrees of second-order coherence

$$g_{\lambda\lambda}^{(2)}(0)=\frac{\langle\tilde{I}_\lambda^2\rangle}{I^2}\ ,\quad \lambda=x,y\quad g_{xy}^{(2)}(0)=\frac{\langle\tilde{I}_x\tilde{I}_y\rangle}{I^2}\ .\qquad (14)$$

We also introduced in (13) the dimensionless parameter η related to the average intensity of the beam, $\eta^2=\langle\tau_x^2+2\tau_y^2\rangle=4\pi e^2I/\omega^2m^2c^3$.

It is worth noting that Eq. (13) agrees with the QED result in the Thomson limit [4]. The correlation functions appearing in QED are of course the quantum mechanical degrees of coherence, instead of the classical quantities (14). It is known, however, that for classical beams both definitions of the degree of coherence yield the same end product. For example, for an incident x-polarized coherent (laser) beam we have $g_{xx}^{(2)}=1$, $g_{yy}^{(2)}=g_{xy}^{(2)}=0$, and this particular case of Eq. (13) agrees with previous classical [11,18] and semiclassical [19] calculations, and with QED [2,10]. It is also instructive to observe that neither potential or superpositions (i) and (ii) of Eq. (1), nor the elliptic potential $\mathbf{A}^{(+)}$, lead to the cross section (13). This result contrasts with the situation previously encountered for the first harmonic cross section (10).

The fluctuations of chaotic light beams, e.g., from thermal sources, can be evaluated by means of the correlation functions (14) and the known [20] ensemble distribution $p(\tilde{I}_\lambda)$ of the instantaneous intensity, $p(\tilde{I}_\lambda)=I_\lambda^{-1}\exp(-\tilde{I}_\lambda/I_\lambda)$. We find

$$g_{xx}^{(2)}(0)=(2I_x^2/I^2)=\tfrac{1}{2}(1+P)^2\ ,$$
$$g_{yy}^{(2)}(0)=(2I_y^2/I^2)=\tfrac{1}{2}(1-P)^2\ ,\qquad (15)$$
$$g_{xy}^{(2)}(0)=(I_xI_y/I^2)=\tfrac{1}{4}(1-P^2)\ ,$$

and $g^{(2)}(0)=I^{-2}\langle\tilde{I}^2\rangle=(3+P^2)/2$ for the total degree of coherence, so that the decomposition [4] $g^{(2)}(0)=g_{xx}^{(2)}(0)+g_{yy}^{(2)}(0)+2g_{xy}^{(2)}(0)$, valid for arbitrary beams, is satisfied. For completely unpolarized light ($P=0$), Eq. (13) goes over, as expected, into the fully unpolarized quantum cross section given in [10]. Some predictions of the second harmonic cross sections at different values of P are shown in Fig. 1 for incident chaotic light of wavelength 100 nm and intensity $\eta=5\times10^{-6}$. It is interesting to contrast this cross section with the prediction of (10) for ordinary Thomson scattering for the same incident beam and values of P (inset of Fig. 1).

The original motivation for this paper was to perform classical calculations of the scattering of partially polarized light by free electrons so as to address the issue of classical-quantum correspondence for higher harmonic radiation. An immediate consequence was to find agreement with QED [4,10], and to correct [11] and clarify [18] previous published results. Before [2], it was also used to clear up a result [21] for linearly polarized light. The upshot is that the correspondence principle is relevant for a consistent overall picture.

A key point in the present discussion is the electromagnetic potential representing a partially polarized wave. The potential must take into account the fact, which is

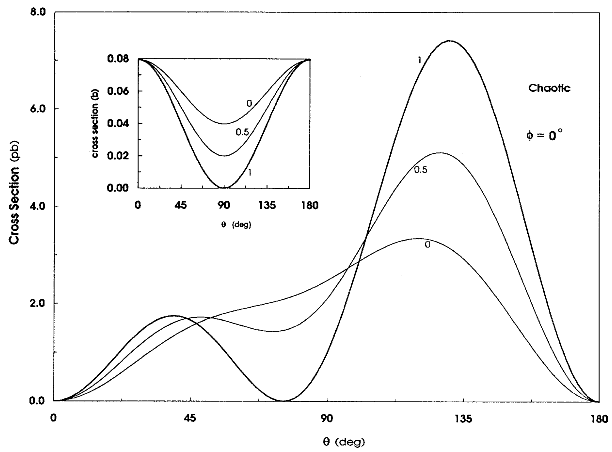

FIG. 1. The Thomson (inset) and second harmonic differential cross sections $d\sigma/d\Omega$ at values of the degree of polarization $P=0$, 0.5, 1, for the azimuthal angle: $\phi=0°$.

nontrivial and very important in this case, that its polarization vector varies with time at all space points of the wave differently than for linear or elliptic polarization although still represented by the superposition of two linearly polarized waves. The expression in Eq. (2) possesses this characteristic. Consequently, as the degree of polarization changes from unpolarized to linearly polarized light, it describes correctly both the statistical properties of the incident beam and also the fundamental and second harmonic radiation emitted by the electron. Any of the simpler linear or elliptic potentials considered above, e.g., Eqs. (1a) and (1b), yields the ordinary Thomson cross section (10a). By contrast, they cannot accommodate the second harmonic cross section. One significant outcome here is that only higher harmonic radiation enables one to extract the potential representing a partially polarized plane wave. We may conclude therefore that the electron motion in the wave represented by expression (2) induces the emission of radiation that, classically and in the Thomson limit, corresponds to the QED Compton-emitted photon for processes in which two or more photons, with distinct or equal polarizations, are simultaneously absorbed.

Although we have focused on the Thomson limit and low-intensity cross sections for a rest electron, it is of great interest to know to what extent the classical-QED correspondence discussed here unfolds as the electron becomes increasingly relativistic and for arbitrary intensities and pulse shapes of the incident beam. It would seem worthwhile to discuss this situation for the higher-order cross sections induced by partially polarized light. Moreover, we can also expect that for arbitrary intensities of the incident beam our choice of initial condition that yields expression (3) should also lead in the Thomson limit to a more general cross section induced by partially polarized high intensity radiation, similar to the Schott cross section [12,13] in the case of incident circularly polarized light. Work in these directions is proceeding.

The magnitude of the second harmonic cross section can be magnified through its nonlinear dependence on the intensity, using larger but realistic values for η [2,4] in the low-intensity regime. Another less familiar mechanism [22] concerns the second-order degrees of coherence defined in (14). For a pulsed beam with a pulse separation equal to T and pulse duration τ_0, the second-order coherence is generally expected to vary as $\sim 1/f$ where $f=\tau_0/T$ with $0<f\leq 1$ [23]. Assuming pulses of sufficiently long duration and low intensity such that the approximations made here are not entirely inadequate, then the special case when $f \ll 1$ should result in a considerable enhancement of the second harmonic cross section.

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